Chapter 3 Matrices

EXERCISE 3.1

Question 1:

In the matrix $A=\begin{pmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{pmatrix}$, write:

(i) The order of the matrix

(ii) The number of elements

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution:

(i) Since, in the given matrix, the number of rows is 3 and the number of columns is 4, the order of the matrix is 3×4.

(ii) Since the order of the matrix is 3×4, there are $3\times 4=12$ elements.

(iii) Here,

$$a_{13}=19$$
$$a_{21}=35$$
$$a_{33}=-5$$
$$a_{24}=12$$
$$a_{23}=\frac{5}{2}$$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution:

We know that if a matrix is of the order $m\times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: $(1,24),(24,1),(2,12),(12,2),(3,8),(8,3),(4,6)$ and $(6,4)$.

Hence, the possible orders of a matrix having 24 elements are:

$$(1\times 24),(24\times 1),(2\times 12),(12\times 2),(3\times 8),(8\times 3),(4\times 6) \text{ and } (6\times 4).$$

$(1,13)$ and $(13,1)$ are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are (1×13) and (13×1).

Question 3:

If a matrix has 18 elements, what are the possible order it can have? What, if it has 5 elements?

Solution:

We know that if a matrix is of the order $m\times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: $(1,18),(18,1),(2,9),(9,2),(3,6)$ and $(6,3)$.

Hence, the possible orders of a matrix having 18 elements are:

$$(1\times18),(18\times1),(2\times9),(9\times2),(3\times6) \text{ and } (6\times3).$$

(1×5) and (5×1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are (1×5) and (5×1).

Question 4:

Construct a 2×2 matrix, $A=[a_{ij}]$, whose elements are given by:

(i) $a_{ij}=\frac{(i+j)^2}{2}$

(ii) $a_{ij}=\frac{i}{j}$

(iii) $a_{ij}=\frac{(i+2j)^2}{2}$

Solution:

In general, a 2×2 matrix is given by $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

(i) $a_{ij}=\frac{(i+j)^2}{2};\ \ i,j=1,2$

Therefore,

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Thus, the required matrix is $A = \begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$

(ii) $a_{ij} = \frac{i}{j}; \quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$

Thus, the required matrix is $A = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}; \quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = 8$$

$$a_{22} = \frac{(2+4)^2}{2} = 18$$

Thus, the required matrix is $A = \begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$

Question 5:

In general, a 3×4 matrix whose elements are given by

(i) $a_{ij} = \frac{1}{2}|-3i + j|$

(ii) $a_{ij} = 2i - j$

Solution:

In general, a 3×4 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$

(i) Given $a_{ij} = \frac{1}{2}|-3i + j|;\ i = 1,2,3\ \ j = 1,2,3,4$

$$a_{11} = \frac{1}{2}|-3(1)+1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$
$$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$$
$$a_{31} = \frac{1}{2}|-3(3)+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$$
$$a_{22} = \frac{1}{2}|-3(2)+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$
$$a_{32} = \frac{1}{2}|-3(3)+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3(1)+3| = \frac{1}{2}|-3+3| = 0$$
$$a_{23} = \frac{1}{2}|-3(2)+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$$
$$a_{33} = \frac{1}{2}|-3(3)+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$$
$$a_{24} = \frac{1}{2}|-3(2)+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$
$$a_{34} = \frac{1}{2}|-3(3)+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$$

Thus, the required matrix is $A=\begin{pmatrix}1 & \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{5}{2} & 2 & \frac{3}{2} & 1\\ 4 & \frac{7}{2} & 3 & \frac{5}{2}\end{pmatrix}$

(ii) $a_{ij}=2i-j;\ i=1,2,3\ \ j=1,2,3,4$

$$a_{11}=2(1)-1=2-1=1$$
$$a_{21}=2(2)-1=4-1=3$$
$$a_{31}=2(3)-1=6-1=5$$

$$a_{12}=2(1)-2=2-2=0$$
$$a_{22}=2(2)-2=4-2=2$$
$$a_{32}=2(3)-2=6-2=4$$

$$a_{13}=2(1)-3=2-3=-1$$
$$a_{23}=2(2)-3=4-3=1$$
$$a_{33}=2(3)-3=6-3=3$$

$$a_{14}=2(1)-4=2-4=-2$$
$$a_{24}=2(2)-4=4-4=0$$
$$a_{34}=2(3)-4=6-4=2$$

Thus, the required matrix is $A=\begin{pmatrix}1 & 0 & -1 & -2\\ 3 & 2 & 1 & 0\\ 5 & 4 & 3 & 2\end{pmatrix}$

Question 6:

Find the value of x, y and z from the following equation:

(i) $\begin{pmatrix}4 & 3\\ x & 5\end{pmatrix}=\begin{pmatrix}y & z\\ 1 & 5\end{pmatrix}$

(ii) $\begin{pmatrix}x+y & 2\\ 5+z & xy\end{pmatrix}=\begin{pmatrix}6 & 2\\ 5 & 8\end{pmatrix}$

(iii) $\begin{pmatrix}x+y+z\\ x+z\\ y+z\end{pmatrix}=\begin{pmatrix}9\\ 5\\ 7\end{pmatrix}$

Solution:

(i) $\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x = 1,\ y = 4$ and $z = 3$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x + y = 6$

$xy = 8$

$5 + z = 5$

Hence,

$\Rightarrow 5 + z = 5$

$\Rightarrow z = 0$

We know that $(a-b)^2 = (a+b)^2 - 4ab$

$\Rightarrow (x-y)^2 = (6)^2 - 8 \times 4$

$\Rightarrow (x-y)^2 = 36 - 32$

$\Rightarrow (x-y)^2 = 4$

$\Rightarrow (x-y) = \pm 2$

Equating $x - y = 2$ and $x + y = 6$, we get $x = 4,\ y = 2$

Similarly, Equating $x - y = -2$ and $x + y = 6$, we get $x = 2,\ y = 4$

Thus, $x = 4,\ y = 2,\ z = 0$ or $x = 2,\ y = 4,\ z = 0$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x + y + z = 9 \quad \ldots(1)$

$x + z = 5 \quad \ldots(2)$

$y + z = 7 \quad \ldots(3)$

From (1) and (2), we have

$$\Rightarrow y+5=9$$
$$\Rightarrow y=4$$

From (3), we have

$$\Rightarrow 4+z=7$$
$$\Rightarrow z=3$$

Therefore,

$$\Rightarrow x+z=5$$
$$\Rightarrow x+3=5$$
$$\Rightarrow x=2$$

Thus, $x=2, y=4, z=3$

Question 7:

Find the value of a, b, c and d from the equation:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$a-b=-1 \quad \ldots(1)$$
$$2a-b=0 \quad \ldots(2)$$
$$2a+c=5 \quad \ldots(3)$$
$$3c+d=13 \quad \ldots(4)$$

From (2),

$$b=2a$$

Putting this value in (1),

$$\Rightarrow a-2a=-1$$
$$\Rightarrow a=1$$

Hence,

$$\Rightarrow b=2$$

Putting $a=1$ in (3),

$$\Rightarrow 2(1)+c=5$$
$$\Rightarrow c=3$$

Putting $c=3$ in (4),

$$\Rightarrow 3(3)+d=13$$
$$\Rightarrow d=4$$

Thus, $a=1, b=2, c=3$ and $d=4$.

Question 8:

$A=\left[a_{ij}\right]_{m\times n}$ is a square matrix, if

(A) $m<n$ (B) $m>n$ (C) $m=n$ (D) None of these

Solution:

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A=\left[a_{ij}\right]_{m\times n}$ is a square matrix, if $m=n$.

Thus, the correct option is C.

Question 9:

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x=\frac{-1}{3}, y=7$ (B) Not possible to find (C) $y=7, x=\frac{-2}{3}$ (D) $x=\frac{-1}{3}, y=\frac{-2}{3}$

Solution:

The given matrices are $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x+7=0 \Rightarrow x=\frac{-7}{3}$$

$$y-2=5 \Rightarrow y=7$$

$$y+1=8 \Rightarrow y=7$$

$$2-3x=4 \Rightarrow x=\frac{-2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Thus, the correct option is B.

Question 10:

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Solution:

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Hence, by the multiplication principle, the required number of possible matrices is $2^9=512$.

Thus, the correct option is D.

EXERCISE 3.2

Question 1:

Let $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$. Find each of the following:

(i) $A+B$

(ii) $A-B$

(iii) $3A-C$

(iv) AB

(v) BA

Solution:

(i) $A+B$

$$\Rightarrow\begin{pmatrix}2 & 4\\3 & 2\end{pmatrix}+\begin{pmatrix}1 & 3\\-2 & 5\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}2+1 & 4+3\\3-2 & 2+5\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}3 & 7\\1 & 7\end{pmatrix}$$

(ii) $A-B$

$$\Rightarrow\begin{pmatrix}2 & 4\\3 & 2\end{pmatrix}-\begin{pmatrix}1 & 3\\-2 & 5\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}2-1 & 4-3\\3+2 & 2-5\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & 1\\5 & -3\end{pmatrix}$$

(iii) $3A-C$

$$\Rightarrow 3\begin{pmatrix}2 & 4\\3 & 2\end{pmatrix}-\begin{pmatrix}-2 & 5\\3 & 4\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}3\times2 & 3\times4\\3\times3 & 3\times2\end{pmatrix}-\begin{pmatrix}-2 & 5\\3 & 4\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}6+2 & 12-5\\9-3 & 6-4\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}8 & 7\\6 & 2\end{pmatrix}$$

(iv) AB

$$\Rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & 26 \\ -1 & 19 \end{pmatrix}$$

(v) BA

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 10 \\ 11 & 2 \end{pmatrix}$$

Question 2:

Compute the following:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}+\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

(ii) $\begin{pmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{pmatrix}+\begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix}+\begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$

(iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix}+\begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

Solution:

(i) $$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a+a & b+b \\ -b+b & a+a \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2a & 2b \\ 0 & 2a \end{pmatrix}$$

(ii) $$\begin{pmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{pmatrix}$$

(iii) $$\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{pmatrix}$$

(iv) $$\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \cos^2 x+\sin^2 x & \sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & \cos^2 x+\sin^2 x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Question 3:

Compute the indicated products:

(i) $$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

(ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{pmatrix}$

(v) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$

(vi) $\begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$

Solution:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & -b(-b)+a(a) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{pmatrix}$$

(ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{pmatrix}$$

(iii) $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{pmatrix}$$

(iv) $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{pmatrix}$$

(v) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{pmatrix}$$

(vi) $\begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 14 & -6 \\ 4 & 5 \end{pmatrix}$$

Question 4:

If $A=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}, B=\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}, C=\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$.

Solution:

$$(A+B)=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}+\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$$
$$=\begin{pmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{pmatrix}$$
$$(B-C)=\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}-\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$$
$$=\begin{pmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{pmatrix}$$

Now,

$$A+(B-C)=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}+\begin{pmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{pmatrix}$$

$$(A+B)-C=\begin{pmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{pmatrix}-\begin{pmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{pmatrix}$$
$$=\begin{pmatrix}0 & 0 & -3\\ 9 & -1 & 5\\ 2 & 1 & 1\end{pmatrix}$$

Hence, $A+(B-C)=(A+B)-C$.

Question 5:

If $A=\begin{pmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{pmatrix}$ and $B=\begin{pmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{pmatrix}$, then compute $3A-5B$.

Solution:

$$3A-5B=3\begin{pmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{pmatrix}-5\begin{pmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{pmatrix}$$

$$=\begin{pmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{pmatrix}-\begin{pmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

Question 6:

Simplify $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$.

Solution:

$$\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{pmatrix}+\begin{pmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Question 7:

Find X and Y, if

(i) $X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}$ and $X-Y=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$

(ii) $2X+3Y=\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix}$ and $3X+2Y=\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix}$

Solution:

(i)

$$X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix} \qquad \ldots(1)$$

$$X-Y=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix} \qquad \ldots(2)$$

Adding equations (1) and (2),

$$2X=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$$

$$=\begin{pmatrix}10 & 0\\ 2 & 8\end{pmatrix}$$

$$X=\frac{1}{2}\begin{pmatrix}10 & 0\\ 2 & 8\end{pmatrix}$$

$$=\begin{pmatrix}5 & 0\\ 1 & 4\end{pmatrix}$$

Now,

$$\Rightarrow X+Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}$$

$$\Rightarrow Y=\begin{pmatrix}7 & 0\\ 2 & 5\end{pmatrix}-\begin{pmatrix}5 & 0\\ 1 & 4\end{pmatrix}$$

$$\Rightarrow Y=\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$$

(ii)

$$2X+3Y=\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix} \qquad \ldots(1)$$

$$3X+2Y=\begin{pmatrix}2 & -2\\ -1 & 5\end{pmatrix} \qquad \ldots(2)$$

Multiplying equation (1) by 2,

$$2(2X+3Y)=2\begin{pmatrix}2 & 3\\ 4 & 0\end{pmatrix}$$

$$4X+6Y=\begin{pmatrix}4 & 6\\ 8 & 0\end{pmatrix} \qquad \ldots(3)$$

Multiplying equation (2) by 3,

$$3(3X+2Y)=3\begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$$

$$9X+6Y=\begin{pmatrix} 6 & -6 \\ -3 & 15 \end{pmatrix} \qquad \ldots(4)$$

From (3) and (4),

$$(4X+6Y)-(9X+6Y)=\begin{pmatrix} 4 & 6 \\ 8 & 0 \end{pmatrix}-\begin{pmatrix} 6 & -6 \\ -3 & 15 \end{pmatrix}$$

$$-5X=\begin{pmatrix} 4-6 & 6+6 \\ 8+3 & 0-15 \end{pmatrix}$$

$$-5X=\begin{pmatrix} -2 & 12 \\ 11 & -15 \end{pmatrix}$$

$$X=\frac{-1}{5}\begin{pmatrix} -2 & 12 \\ 11 & -15 \end{pmatrix}$$

$$=\begin{pmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{pmatrix}$$

Now

$$\Rightarrow 2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$$

$$\Rightarrow 2\begin{pmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{pmatrix} + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{pmatrix} + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$$

$$\Rightarrow 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} - \begin{pmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{pmatrix}$$

$$\Rightarrow 3Y = \begin{pmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{pmatrix}$$

$$\Rightarrow Y = \frac{1}{3}\begin{pmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{pmatrix}$$

$$\Rightarrow Y = \begin{pmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{pmatrix}$$

Question 8:

Find X, if $Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$.

Solution:

$$2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\Rightarrow 2X + \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\Rightarrow 2X = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$\Rightarrow 2X = \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$

Question 9:

Find x and y, if $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$.

Solution:

$$\Rightarrow 2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 6 \\ 0 & 2x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2+y & 6 \\ 1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2 + y = 5$$
$$\Rightarrow y = 3$$

$$2x + 2 = 8$$
$$\Rightarrow x = 3$$

Therefore, $x = 3$ and $y = 3$.

Question 10:

Solve the equation for x, y, z and t if $2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$.

Solution:

$$\Rightarrow 2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} 2x & 2z \\ 2y & 2t \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x+3=9$$
$$\Rightarrow 2x=6$$
$$\Rightarrow x=3$$

$$2y=12$$
$$\Rightarrow y=6$$

$$2z-3=15$$
$$\Rightarrow 2z=18$$
$$\Rightarrow z=9$$

$$2t+6=18$$
$$\Rightarrow 2t=12$$
$$\Rightarrow t=6$$

Therefore, $x=3, y=6, z=9$ and $t=6$.

Question 11:

If $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, find values of x and y.

Solution:

$$\Rightarrow x\begin{pmatrix}2\\3\end{pmatrix}+y\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2x\\3x\end{pmatrix}+\begin{pmatrix}-y\\y\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2x-y\\3x+y\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x-y=10 \qquad \ldots(1)$$

$$3x+y=5 \qquad \ldots(2)$$

By adding these two equations, we get

$$5x=15$$

$$\Rightarrow x=3$$

Now, putting this value in (2)

$$\Rightarrow 3x+y=5$$

$$\Rightarrow y=5-3x$$

$$\Rightarrow y=5-3(3)$$

$$\Rightarrow y=5-9$$

$$\Rightarrow y=-4$$

Therefore, $x=3$ and $y=4$.

Question 12:

Given $3\begin{pmatrix}x & y\\ z & w\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2w\end{pmatrix}+\begin{pmatrix}4 & x+y\\ z+w & 3\end{pmatrix}$, find values of w, x, y and z.

Solution:

$$\Rightarrow 3\begin{pmatrix}x & y\\ z & w\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2w\end{pmatrix}+\begin{pmatrix}4 & x+y\\ z+w & 3\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}3x & 3y\\ 3z & 3w\end{pmatrix}=\begin{pmatrix}x+4 & 6+x+y\\ -1+z+w & 2w+3\end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$\Rightarrow 3x = x + 4$
$\Rightarrow 2x = 4$
$\Rightarrow x = 2$

$\Rightarrow 3y = 6 + x + y$
$\Rightarrow 2y = 6 + x$
$\Rightarrow 2y = 6 + 2$
$\Rightarrow 2y = 8$
$\Rightarrow y = 4$

$\Rightarrow 3w = 2w + 3$
$\Rightarrow w = 3$

$\Rightarrow 3z = -1 + z + w$
$\Rightarrow 2z = w - 1$
$\Rightarrow 2z = 3 - 1$
$\Rightarrow 2z == 2$
$\Rightarrow z = 1$

Therefore, $x = 2, y = 4, z = 1$ and $w = 3$

Question 13:

If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that $F(x)F(y) = F(x+y)$.

Solution:

It is given that $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then, $F(y) = \begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Now,

$$F(x+y) = \begin{pmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$F(x)F(y)=\begin{pmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$=\begin{pmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$=F(x+y)$$

Therefore, $F(x)F(y)=F(x+y)$

Question 14:

Show that

(i) $\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\neq\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}$

(ii) $\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{pmatrix}\neq\begin{pmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{pmatrix}$

Solution:

(i) $\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\neq\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}$

$$\begin{pmatrix}5 & -1\\ 6 & 7\end{pmatrix}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}=\begin{pmatrix}5(2)-1(3) & 5(1)-1(4)\\ 6(2)+7(3) & 6(1)+7(4)\end{pmatrix}$$

$$=\begin{pmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{pmatrix}$$

$$=\begin{pmatrix}7 & 1\\ 33 & 34\end{pmatrix}$$

$$\begin{pmatrix}2&1\\3&4\end{pmatrix}\begin{pmatrix}5&-1\\6&7\end{pmatrix}=\begin{pmatrix}2(5)+1(6)&2(-1)+1(7)\\3(5)+4(6)&3(-1)+4(7)\end{pmatrix}$$

$$=\begin{pmatrix}10+6&-2+7\\15+24&-3+28\end{pmatrix}$$

$$=\begin{pmatrix}16&5\\39&25\end{pmatrix}$$

Thus, $\begin{pmatrix}5&-1\\6&7\end{pmatrix}\begin{pmatrix}2&1\\3&4\end{pmatrix}\neq\begin{pmatrix}2&1\\3&4\end{pmatrix}\begin{pmatrix}5&-1\\6&7\end{pmatrix}$

(ii) $\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}\neq\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}$

$$\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}=\begin{pmatrix}1(-1)+2(0)+3(2)&1(1)+2(-1)+3(3)&1(0)+2(1)+3(4)\\0(-1)+1(0)+0(2)&0(1)+1(-1)+0(3)&0(0)+1(1)+0(4)\\1(-1)+1(0)+0(2)&1(1)+1(-1)+0(3)&1(0)+1(1)+0(4)\end{pmatrix}$$

$$=\begin{pmatrix}5&8&14\\0&-1&1\\-1&0&1\end{pmatrix}$$

$$\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}=\begin{pmatrix}-1(1)+1(0)+0(1)&-1(2)+1(1)+0(1)&-1(3)+1(0)+0(0)\\0(1)+(-1)(0)+1(1)&0(2)+(-1)(1)+1(1)&0(3)+-1(0)+1(0)\\2(1)+3(0)+4(1)&2(2)+3(1)+4(1)&2(3)+3(0)+4(0)\end{pmatrix}$$

$$=\begin{pmatrix}-1&-1&-3\\1&0&0\\6&11&6\end{pmatrix}$$

Thus, $\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}\neq\begin{pmatrix}-1&1&0\\0&-1&1\\2&3&4\end{pmatrix}\begin{pmatrix}1&2&3\\0&1&0\\1&1&0\end{pmatrix}$

Question 15:

Find $A^2-5A+6I$, if $A=\begin{pmatrix}2&0&1\\2&1&3\\1&-1&0\end{pmatrix}$

Solution:

$A^2 = A.A$

$$= \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+1(1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{pmatrix}$$

$$= \begin{pmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$

Therefore,

$$A^2 - 5A + 6I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 6\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{pmatrix}$$

Question 16:

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

Solution:

$A^2 = A.A$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$$

Now,

$A^3 = A^2.A$

$$= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$$

Therefore,

$$A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - 6\begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix}$$

$$=\begin{pmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{pmatrix}-\begin{pmatrix}30 & 0 & 48\\12 & 24 & 30\\48 & 0 & 78\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}=0$$

Hence, $A^3-6A^2+7A+2I=0$.

Question 17:

If $A=\begin{pmatrix}3 & -2\\4 & -2\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, find k so that $A^2=kA-2I$.

Solution:

$A^2 = A.A$

$$=\begin{pmatrix}3 & -2\\4 & -2\end{pmatrix}\begin{pmatrix}3 & -2\\4 & -2\end{pmatrix}$$

$$=\begin{pmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{pmatrix}$$

$$=\begin{pmatrix}1 & -2\\4 & -4\end{pmatrix}$$

Now,

$$\Rightarrow A^2=kA-2I$$

$$\Rightarrow \begin{pmatrix}1 & -2\\4 & -4\end{pmatrix}=k\begin{pmatrix}3 & -2\\4 & -2\end{pmatrix}-2\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & -2\\4 & -4\end{pmatrix}=\begin{pmatrix}3k & -2k\\4k & -2k\end{pmatrix}-\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & -2\\4 & -4\end{pmatrix}=\begin{pmatrix}3k-2 & -2k\\4k & -2k-2\end{pmatrix}$$

Comparing the corresponding elements, we have:

$$3k-2=1$$

$$\Rightarrow 3k=3$$

$$\Rightarrow k=1$$

Therefore, the value of $k=1$.

Question 18:

If $A=\begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and I is the identity matrix of order 2, show that $I+A=(I-A)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Solution:

$LHS = I + A$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{pmatrix} \qquad \ldots(1)$$

$$RHS=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\begin{pmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{pmatrix}\right)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}\\ -\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\begin{pmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-\tan\frac{\alpha}{2}\\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{pmatrix}$$

$$=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix} \qquad \ldots(2)$$

Thus, from (1) and (2), we get

$$I+A=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

Question 19:

A trust fund has ₹30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(i) ₹ 1800

(ii) ₹ 2000

Solution:

(i) Let ₹x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹$(30000-x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of ₹1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100}\right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹15000 in the first bond and the remaining ₹15000 in the second bond.

(ii) Let ₹x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹$(30000-x)$.

Therefore, in order to obtain an annual total interest of ₹2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹5000 in the first bond and the remaining ₹25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution:

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}=12\left[10(80)+8(60)+10(40)\right]$$

$$=12(800+480+400)$$

$$=12(1680)$$

$$=20160$$

Thus, the bookshop will receive ₹20160 from the sale of all these books.

Question 21:

Assume X, Y, Z, W and P are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ and $p\times k$ respectively. The restriction on n, k and p so that $PY+WY$ will be defined are:

(A) $k=3, p=n$

(B) k is arbitrary, $p=2$

(C) p is arbitrary, $k=3$

(D) $k=2, p=3$

Solution:

Matrices P and Y are of the orders $p\times k$ and $3\times k$ respectively.
Therefore, matrix PY will be defined if $k=3$.
Consequently, PY will be of the order $p\times k$.
Matrices W and Y are of the orders $n\times 3$ and $3\times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order $n\times k$.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p\times k$ and WY is of the order $n\times k$.
Therefore, we must have $p=n$.

Thus, $k=3$ and $p=n$ are the restrictions on n, k and p so that $PY+WY$ will be defined.

The correct option is A.

Question 22:

Assume X, Y, Z, W and P are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ and $p\times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(A) $p\times 2$
(B) $2\times n$
(C) $n\times 3$
(D) $p\times n$

Solution:

Matrix X is of the order $2\times n$.
Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2\times p$, i.e., $2\times n$ [Since $n = p$]
Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and 5Z are of the order $2\times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2\times n$.

The correct option is B.

EXERCISE 3.3

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

Solution:

(i) Let $A = \begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 5 & \frac{1}{2} & -1 \end{pmatrix}$

(ii) Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

(iii) Let $A = \begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{pmatrix}$

Question 2:

If $A=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

Solution:

It is given that $A=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}$

Hence, we have $A'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}$ and $B'=\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$

(i) $(A+B)=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}+\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{pmatrix}$

Hence,

$$(A+B)'=\begin{pmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{pmatrix}$$

Now,

$$A'+B'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}+\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$$
$$=\begin{pmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{pmatrix}$$

Thus, $(A+B)'=A'+B'$.

(ii) $$(A-B)=\begin{pmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{pmatrix}-\begin{pmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{pmatrix}$$

Hence,

$$(A-B)'=\begin{pmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{pmatrix}$$

Now,

$$A'-B'=\begin{pmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{pmatrix}-\begin{pmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{pmatrix}$$

Thus, $(A-B)'=A'-B'$.

Question 3:

If $A'=\begin{pmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{pmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

Solution:

It is known that $A=(A')'$

Hence,

$$A=\begin{pmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{pmatrix} \text{ and } B'=\begin{pmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{pmatrix}$$

(i) $$A+B=\begin{pmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{pmatrix}+\begin{pmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{pmatrix}=\begin{pmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{pmatrix}$$

Therefore,

$$(A+B)' = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

Now,

$$A'+B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

Hence, $(A+B)' = A' + B'$.

(ii) $$A - B = \begin{pmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{pmatrix}$$

Therefore,

$$(A-B)' = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Now,

$$A' - B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Hence, $(A-B)' = A' - B'$.

Question 4:

If $A' = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$, then find $(A+2B)'$.

Solution:

It is known that $A = (A')'$.
Therefore,

$$A = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$$

Now,

$$A+2B=\begin{pmatrix}-2 & 1\\ 3 & 2\end{pmatrix}+2\begin{pmatrix}-1 & 0\\ 1 & 2\end{pmatrix}$$
$$=\begin{pmatrix}-2 & 1\\ 3 & 2\end{pmatrix}+\begin{pmatrix}-2 & 0\\ 2 & 4\end{pmatrix}$$
$$=\begin{pmatrix}-4 & 1\\ 5 & 6\end{pmatrix}$$

Question 5:

For the matrices A and B, verify that $(AB)'=B'A'$ where

(i) $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}, B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Solution:

(i) It is given that $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

Hence,

$$AB=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$$
$$=\begin{bmatrix}-1 & 2 & 1\\ 4 & -8 & -4\\ -3 & 6 & 3\end{bmatrix}$$

Therefore,

$$(AB)'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$

Now,

$A'=\begin{bmatrix}1 & -4 & 3\end{bmatrix}$ and $B'=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$

Hence,

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Thus, $(AB)' = B'A'$

(ii) It is given that $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Hence,

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

Therefore,

$$(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now,

$A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$ and $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

Therefore,

$$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Thus, $(AB)' = B'A'$.

Question 6:

If

(i) $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, then verify $A'A = I$

(ii) $A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$, then verify $A'A = I$

Solution:

(i) It is given that $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Now,

$$\begin{aligned} A'A &= \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \\ &= \begin{pmatrix} \cos\alpha\cos\alpha + (-\sin\alpha)(-\sin\alpha) & \sin\alpha\cos\alpha + (-\sin\alpha)\cos\alpha \\ \sin\alpha\cos\alpha + \cos\alpha(-\sin\alpha) & \sin\alpha\sin\alpha + \cos\alpha\cos\alpha \end{pmatrix} \\ &= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= I \end{aligned}$$

Thus, $A'A = I$

(ii) It is given that $A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}$$

Now,

$$A'A = \begin{pmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}\begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \sin\alpha\sin\alpha + (-\cos\alpha)(-\cos\alpha) & \sin\alpha\cos\alpha + (-\cos\alpha)\sin\alpha \\ \sin\alpha\cos\alpha + \sin\alpha(-\cos\alpha) & \sin\alpha\sin\alpha + \cos\alpha\cos\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$= I$$

Thus, $A'A = I$

Question 7:

(i) Show that the matrix $A = \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ is a skew symmetric matrix.

Solution:

(i) $A = \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$

Now,

$$A' = \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$$
$$= A$$

Hence, A is a symmetric matrix.

(ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

$$A' = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= -A$$

Hence, A is a skew symmetric matrix.

Question 8:

For the matrix $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew symmetric matrix.

Solution:

It is given that $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$

Hence, $A' = \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$

(i) $(A + A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$

Therefore,

$$(A + A')' = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$$

$$= (A + A')$$

Thus, $(A + A')$ is a symmetric matrix.

(ii) $(A - A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Therefore,

$$(A - A')' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= -(A - A')$$

Thus, $(A-A')$ is a skew symmetric matrix.

Question 9:

Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$.

Solution:

It is given that $A=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$

Hence,

$$A'=\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}+\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore,

$$\frac{1}{2}(A+A')=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$(A-A')=\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}-\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{pmatrix}$$

Thus,

$$\frac{1}{2}(A-A')=\begin{pmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{pmatrix}$$

Question 10:

Express the following as the sum of a symmetric and skew symmetric matrix:

(i) $\begin{pmatrix}3 & 5\\ 1 & -1\end{pmatrix}$

(ii) $\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$

(iii) $\begin{pmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{pmatrix}$

(iv) $\begin{pmatrix}1 & 5\\ -1 & 2\end{pmatrix}$

Solution:

(i) Let $A=\begin{pmatrix}3 & 5\\ 1 & -1\end{pmatrix}$

Hence,

$$A'=\begin{pmatrix}3 & 1\\ 5 & -1\end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix}3 & 5\\ 1 & -1\end{pmatrix}+\begin{pmatrix}3 & 1\\ 5 & -1\end{pmatrix}$$
$$=\begin{pmatrix}6 & 6\\ 6 & -2\end{pmatrix}$$

Let

$$P = \frac{1}{2}(A + A')$$
$$= \frac{1}{2}\begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}$$

Now,

$$P' = \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}$$
$$= P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,

$$(A - A') = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$

Let

$$Q = \frac{1}{2}(A - A')$$
$$= \frac{1}{2}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

Now,

$$Q' = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$
$$= -Q$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{pmatrix}3 & 3\\3 & -1\end{pmatrix}+\begin{pmatrix}0 & 2\\-2 & 0\end{pmatrix}$$
$$=\begin{pmatrix}3 & 5\\1 & -1\end{pmatrix}$$
$$=A$$

(ii) Let $A=\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$

Hence,

$$A'=\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}+\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}=\begin{pmatrix}12 & -4 & 4\\-4 & 6 & -2\\4 & -2 & 6\end{pmatrix}$$

Let

$$P=\frac{1}{2}(A+A')$$
$$=\frac{1}{2}\begin{pmatrix}12 & -4 & 4\\-4 & 6 & -2\\4 & -2 & 6\end{pmatrix}$$
$$=\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$$

Now,

$$P'=\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$$
$$=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

$$(A-A')=\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}-\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}$$
$$=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

Let

$$Q=\frac{1}{2}(A-A')$$
$$=\frac{1}{2}\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$
$$=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

Now,

$$Q'=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$
$$=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}+\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$
$$=\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}$$
$$=A$$

(iii) Let $A=\begin{pmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{pmatrix}$

Hence,

$$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

Now,

$$\left(A + A'\right) = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix}$$

Let

$$P = \frac{1}{2}\left(A + A'\right)$$

$$= \frac{1}{2}\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}$$

Now,

$$P' = \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}$$

$$= P$$

Thus, $P = \frac{1}{2}\left(A + A'\right)$ is a symmetric matrix.

Now,

$$\left(A-A'\right)=\begin{pmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{pmatrix}-\begin{pmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{pmatrix}$$
$$=\begin{pmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{pmatrix}$$

Let

$$Q=\frac{1}{2}\left(A-A'\right)$$
$$=\frac{1}{2}\begin{pmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ \frac{-5}{2} & 0 & 3\\ \frac{-3}{2} & -3 & 0\end{pmatrix}$$

Now,

$$Q'=\begin{pmatrix}0 & \frac{-5}{2} & \frac{-3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{pmatrix}$$
$$=-Q$$

Thus, $Q=\frac{1}{2}\left(A-A'\right)$ is a skew symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix}+\begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

$$=A$$

(iv) Let $A=\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

Hence,

$$A'=\begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$$

Now,

$$(A+A')=\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}+\begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix}$$

Let

$$P=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

Now,

$$P'=\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

$$=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

$$(A-A')=\begin{pmatrix}1 & 5\\ -1 & 2\end{pmatrix}-\begin{pmatrix}1 & -1\\ 5 & 2\end{pmatrix}$$
$$=\begin{pmatrix}0 & 6\\ -6 & 0\end{pmatrix}$$

Let

$$Q=\frac{1}{2}(A-A')$$
$$=\frac{1}{2}\begin{pmatrix}0 & 6\\ -6 & 0\end{pmatrix}$$
$$=\begin{pmatrix}0 & 3\\ -3 & 0\end{pmatrix}$$

Now,

$$Q'=\begin{pmatrix}0 & -3\\ 3 & 0\end{pmatrix}$$
$$=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{pmatrix}1 & 2\\ 2 & 2\end{pmatrix}+\begin{pmatrix}0 & 3\\ -3 & 0\end{pmatrix}$$
$$=\begin{pmatrix}1 & 5\\ -1 & 2\end{pmatrix}$$
$$=A$$

Question 11:

If A,B are symmetric matrices of the same order, then $AB-BA$ is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

Solution:

If A and B are symmetric matrices of the same order, then

$$A'=A \text{ and } B'=B \qquad \ldots(1)$$

Now consider,

$$\begin{aligned}(AB-BA)' &= (AB)'-(BA)' && \left[\because (A-B)' = A'-B'\right]\\ &= B'A'-A'B && \left[\because (AB)' = B'A'\right]\\ &= BA-AB && \left[from\ (1)\right]\\ &= -(AB-BA)\end{aligned}$$

Therefore,

$$(AB-BA)' = -(AB-BA)$$

Thus, $AB-BA$ is a skew symmetric matrix.
The Correct option is A.

Question 12:

If $A = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, then $A+A' = I$, if the value of α is:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

Solution:

It is given that $A = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$

Hence,

$$A' = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}$$

Now,

$$A+A' = I$$

Therefore,

$$\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix} + \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$\Rightarrow 2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{1}{2}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Thus, the correct option is B.

EXERCISE 3.4

Question 1:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} A \qquad \left(R_2 \to \frac{1}{5} R_2\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$$

Question 2:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A \qquad (R_2 \to R_2 - R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

Question 3:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix}$$

Question 4:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & \frac{-1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-5}{2} & 1 \end{pmatrix} A \qquad \left(R_2 \to R_2 - 5R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{-1}{2} \end{pmatrix} = \begin{pmatrix} -7 & 3 \\ \frac{-5}{2} & 1 \end{pmatrix} A \qquad \left(R_1 \to R_1 + 3R_2\right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} A \qquad \left(R_2 \to -2R_1\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix}$$

Question 5:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-7}{2} & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - 7R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ \frac{-7}{2} & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} A \qquad (R_2 \to 2R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}$$

Question 6:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{-1}{2} & 1 \end{pmatrix} A \qquad (R_2 \to R_2 - R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ \frac{-1}{2} & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - 5R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} A \qquad (R_2 \to 2R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

Question 7:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \qquad (C_1 \to C_1 - 2C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} \qquad (C_2 \to C_2 - C_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

Question 8:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}A$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix}A \qquad (R_2 \to R_2 - 3R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}$$

Question 9:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

Question 10:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \qquad (C_1 \to C_1 + 2C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = A\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \qquad (C_2 \to C_2 + C_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix} \qquad \left(C_2 \to \frac{1}{2}C_2\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$$

Question 11:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix} = A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \qquad (C_2 \to C_2 + 3C_1)$$

$$\Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} -2 & 3 \\ -1 & 1 \end{pmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A\begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix} \qquad \left(C_1 \to \frac{1}{2}C_1\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix}$$

Question 12:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$, if exists.

Solution:

Let $A=\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{-1}{2} \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to \frac{1}{6} R_1\right)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{pmatrix} A \qquad \left(R_2 \to R_2 + 2R_1\right)$$

In the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Thus, A^{-1} does not exist.

Question 13:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A=\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} A \qquad (R_2 \to R_2 + R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

Question 14:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 0 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix} A \qquad \left(R_1 \to R_1 - \frac{1}{2} R_2\right)$$

In the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Thus, A^{-1} does not exist.

Question 15:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A=\begin{pmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{pmatrix}$

We know that $A=IA$

Therefore,

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$$

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 0 & 5 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad (R_2\to R_2-R_1)$$

$$\Rightarrow\begin{pmatrix}2 & -3 & 3\\ 0 & 1 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad \left(R_2\to \frac{1}{5}R_2\right)$$

$$\Rightarrow\begin{pmatrix}-1 & -1 & 1\\ 0 & 1 & 0\\ 3 & -2 & 2\end{pmatrix}=\begin{pmatrix}1 & 0 & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1\end{pmatrix}A \qquad (R_1\to R_1-R_3)$$

$$\Rightarrow\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ 3 & 0 & 2\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & \frac{1}{5} & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ \frac{-2}{5} & \frac{2}{5} & 1\end{pmatrix}A \qquad (R_1\to R_1+R_2 \text{ and } R_3\to R_3+2R_2)$$

$$\Rightarrow\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 5\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & \frac{1}{5} & -1\\ \frac{-1}{5} & \frac{1}{5} & 0\\ 2 & 1 & -2\end{pmatrix}A \qquad (R_3\to R_3+3R_1)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left(R_3 \to \frac{1}{5} R_3 \right)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & 0 & \frac{-3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left(R_1 \to R_1 - R_3 \right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \qquad \left(R_1 \to (-1) R_1 \right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix}$$

Question 16:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\Rightarrow\begin{pmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$$

$$\Rightarrow\begin{pmatrix}1 & 3 & -2\\ 0 & 9 & -11\\ 0 & -1 & 4\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -2 & 0 & 1\end{pmatrix}A \qquad (R_2\to R_2+3R_1 \text{ and } R_3\to R_3-2R_1)$$

$$\Rightarrow\begin{pmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & -1 & 4\end{pmatrix}=\begin{pmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ -2 & 0 & 1\end{pmatrix}A \qquad (R_1\to R_1+3R_3 \text{ and } R_2\to R_2+8R_3)$$

$$\Rightarrow\begin{pmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & 0 & 25\end{pmatrix}=\begin{pmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ -15 & 1 & 9\end{pmatrix}A \qquad (R_3\to R_3+R_2)$$

$$\Rightarrow\begin{pmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25}\end{pmatrix}A \qquad \left(R_3\to\frac{1}{25}R_3\right)$$

$$\Rightarrow\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & \frac{-2}{5} & \frac{-3}{5}\\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25}\\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25}\end{pmatrix}A \qquad (R_1\to R_1-10R_3 \text{ and } R_2\to R_2-21R_3)$$

$$\Rightarrow A^{-1}=\begin{pmatrix}1 & \frac{-2}{5} & \frac{-3}{5}\\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25}\\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25}\end{pmatrix}$$

Question 17:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{pmatrix}$, if exists.

Solution:

Let $A=\begin{pmatrix}2&0&-1\\5&1&0\\0&1&3\end{pmatrix}$

We know that $A=IA$

Therefore,

$$\Rightarrow\begin{pmatrix}2&0&-1\\5&1&0\\0&1&3\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}A$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\5&1&0\\0&1&3\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{pmatrix}A\qquad\left(R_1\to\frac{1}{2}R_1\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&1&3\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\0&0&1\end{pmatrix}A\qquad\left(R_2\to R_2-5R_1\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&\frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\\frac{5}{2}&-1&1\end{pmatrix}A\qquad\left(R_3\to R_3-R_2\right)$$

$$\Rightarrow\begin{pmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&1\end{pmatrix}=\begin{pmatrix}\frac{1}{2}&0&0\\\frac{-5}{2}&1&0\\5&-2&2\end{pmatrix}A\qquad\left(R_3\to 2R_3\right)$$

$$\Rightarrow\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{pmatrix}A\qquad\left(R_1\to R_1+\frac{1}{2}R_3\ and\ R_2\to R_2-\frac{5}{2}R_3\right)$$

$$\Rightarrow A^{-1}=\begin{pmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{pmatrix}$$

Question 18:

Matrices A and B will be the inverse of each other only if:

(A) $AB = BA$

(B) $AB = BA = 0$

(C) $AB = 0, BA = I$

(D) $AB = BA = I$

Solution:

We know that if A is a square matrix of order m, and
if there exists another square matrix B of the same order m,
such that $AB = BA = I$, then B is said to be the inverse of A.

In this case, it is clear that A is the inverse of B.

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

The correct option is D.

MISCELLANEOUS EXERCISE

Question 1:

Let $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$, show that $(aI+bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in N$.

Solution:

It is given that $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n=1$, we have:

$$P(1):(aI+bA)=aI+ba^0A=aI+bA$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$

That is, $P(k):(aI+bA)^k = a^kI+ka^{k-1}bA$

Now, we have to prove that the result is true for $n=k+1$.

Consider,

$$\begin{aligned}(aI+bA)^{k+1} &= (aI+bA)^k(aI+bA)\\ &= (a^kI+ka^{k-1}bA)(aI+bA)\\ &= a^{k+1}I+ka^kbAI+a^kbIA+ka^{k-1}b^2A^2\\ &= a^{k+1}I+(k+1)a^kbA+ka^{k-1}b^2A^2 \qquad \ldots(1)\end{aligned}$$

Now,

$$A^2=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=0$$

From (1), we have

$$\begin{aligned}(aI+bA)^{k+1} &= a^{k+1}I+(k+1)a^kbA+0\\ &= a^{k+1}I+(k+1)a^kbA\end{aligned}$$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction, we have:

$(aI+bA)^n = a^n I + na^{n-1}bA$ where $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, n\in N$

Question 2:

If $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$, prove that $A^n=\begin{pmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{pmatrix}, n\in N$.

Solution:

It is given that $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.
For $n=1$, we have:

$$P(1): \begin{pmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{pmatrix} = \begin{pmatrix}3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\end{pmatrix} = \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} = A$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$$P(k): A^k = \begin{pmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\end{pmatrix}$$

Now, we have to prove that the result is true for $n=k+1$.

Since,

$$A^{k+1} = A.A^k$$
$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$$
$$= \begin{pmatrix} 3\cdot3^{k-1} & 3\cdot3^{k-1} & 3\cdot3^{k-1} \\ 3\cdot3^{k-1} & 3\cdot3^{k-1} & 3\cdot3^{k-1} \\ 3\cdot3^{k-1} & 3\cdot3^{k-1} & 3\cdot3^{k-1} \end{pmatrix}$$
$$= \begin{pmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{pmatrix}$$

Therefore, the result is true for $n = k+1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$$

Question 3:

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, prove that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$, where n is any positive integer.

Solution:

It is given that $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): A^1 = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$$
$$= \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$$
$$= A$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$$P(k): A^k = \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix}, n \in N$$

Now, we have to prove that the result is true for $n=k+1$.

Since,

$$\begin{aligned} A^{k+1} &= A.A^k \\ &= \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1-2k & -4k-1(1-2k) \end{pmatrix} \\ &= \begin{pmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix} \\ &= \begin{pmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{pmatrix} \\ &= \begin{pmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{pmatrix} \end{aligned}$$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}; n \in N$$

Question 4:

If A and B are symmetric matrices, prove that $AB-BA$ is a skew symmetric matrix.

Solution:

It is given that A and B are symmetric matrices.
Therefore, we have:

$$A' = A \text{ and } B' = B \qquad \ldots(1)$$

Now,

$$(AB-BA)' = (AB)' - (BA)' \qquad \left[(A-B)' = A' - B'\right]$$
$$= B'A' - A'B' \qquad \left[(AB)' = B'A'\right]$$
$$= BA - AB \qquad \left[\text{Using } (1)\right]$$
$$= -(AB-BA)$$

Hence,

$$(AB-BA)' = -(AB-BA)$$

Thus, $AB-BA$ is a skew symmetric matrix.

Question 5:

Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution:

We suppose that A is a symmetric matrix, then

$$A' = A \qquad \ldots(1)$$

Consider,

$$(B'AB)' = \{B'(AB)\}'$$
$$= (AB)'(B')' \qquad \left[\because (AB)' = B'A'\right]$$
$$= B'A'(B) \qquad \left[\because (B')' = B\right]$$
$$= B'(A'B)$$
$$= B'(AB) \qquad \left[\text{Using } (1)\right]$$

Therefore,

$$(B'AB)' = B'AB$$

Thus, if A is symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew symmetric matrix, then

$$A' = -A \qquad \ldots(2)$$

Consider,

$$\begin{aligned}(B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B')' \\ &= (B'A')B \\ &= B'(-A)B \qquad\qquad [\text{Using } (2)] \\ &= -B'AB\end{aligned}$$

Therefore,

$$(B'AB)' = -B'AB$$

Thus, if A is a skew symmetric matrix, then $B'AB$ is a skew symmetric matrix.

Hence, if A is symmetric or skew symmetric matrix, then $B'AB$ is symmetric or skew symmetric accordingly.

Question 6:

Find the values of x, y, z if the matrix $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfy the equation $A'A = I$.

Solution:

It is given that $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$

Therefore,

$$A' = \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix}$$

Now, $A'A = I$

Hence,

$$\Rightarrow \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix}\begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1$$
$$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$6y^2 = 1$$
$$\Rightarrow y = \pm\frac{1}{\sqrt{6}}$$

$$3z^2 = 1$$
$$\Rightarrow z = \pm\frac{1}{\sqrt{3}}$$

Thus, $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

Question 7:

For what values of $x : \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

Solution:

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Hence,

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \left[6(0)+2(2)+4(x) \right] = 0$$

$$\Rightarrow [4+4x] = 0$$

$$\Rightarrow 4x = -4$$

$$\Rightarrow x = -1$$

Thus, the required value of $x = -1$.

Question 8:

If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 7I = 0$

Solution:

It is given that $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

Therefore,

$$\begin{aligned} A^2 &= A.A \\ &= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{pmatrix} \\ &= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} \end{aligned}$$

Now,

$$\begin{aligned} LHS &= A^2 - 5A + 7I \\ &= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\ &= \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\ &= 0 \\ &= RHS \end{aligned}$$

Thus, $A^2 - 5A + 7I = 0$

Question 9:

Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Solution:

We have

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Hence,

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \left[x(x-2) - 40 + 2x - 8 \right] = 0$$

$$\Rightarrow \left[x^2 - 2x - 40 + 2x - 8 \right] = 0$$

$$\Rightarrow \left[x^2 - 48 \right] = 0$$

$$\Rightarrow x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Thus, $x = \pm 4\sqrt{3}$.

Question 10:

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	**Products**		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit.

Solution:

(a) The unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively.

Consequently, the total revenue in market I can be represented in the form of a matrix as:

$$\begin{aligned} \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} &= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00 \\ &= 25000 + 3000 + 18000 \\ &= 46000 \end{aligned}$$

The total revenue in market II can be represented in the form of a matrix as:

$$\begin{aligned} \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} &= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00 \\ &= 15000 + 30000 + 8000 \\ &= 53000 \end{aligned}$$

Thus, the total revenue in market I is ₹46000 and the total revenue in market II is ₹ 53000.

(b) The unit costs of x, y and z are ₹2.00, ₹1.00 and 50 paise respectively.

Consequently, the total cost prices of all the products in market I can be represented in the form of a matrix as:

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix} = 10000\times 2.00 + 2000\times 1.00 + 18000\times 0.50$$
$$= 20000 + 2000 + 9000$$
$$= 31000$$

Since the total revenue in market I is ₹ 46000, the gross profit in this market in ₹ is

$$46000 - 31000 = 15000$$

The total cost prices of all the products in market II can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix} = 6000\times 2.00 + 20000\times 1.00 + 8000\times 0.50$$
$$= 12000 + 20000 + 4000$$
$$= 36000$$

Since the total revenue in market I is ₹ 53000, the gross profit in this market in ₹ is

$$53000 - 36000 = 17000$$

Thus, the gross profit in market I is ₹ 15000 and in market II is ₹ 17000.

Question 11:

Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

Solution:

It is given that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix.

Therefore, X has to be a 2×2 matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore,

$$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c=7 \quad 2a+5c=-8 \quad 3a+6c=-9a$$
$$b+4d=2 \quad 2b+5d=4 \quad 3b+6d=6$$

Now,

$$a+4c=-7$$
$$\Rightarrow a=-7-4c$$

Therefore,

$$2a+5c=-8$$
$$\Rightarrow 2(-7-4c)+5c=-8$$
$$\Rightarrow -14-8c+5c=-8$$
$$\Rightarrow -3c=6$$
$$\Rightarrow c=-2$$

Hence,

$$\Rightarrow a=-7-4(-2)$$
$$\Rightarrow a=-7+8$$
$$\Rightarrow a=1$$

Now,

$$b+4d=2$$
$$\Rightarrow b=2-4d$$

Therefore,

$$2b+5d=4$$
$$\Rightarrow 2(2-4d)+5d=4$$
$$\Rightarrow 4-8d+5d=4$$
$$\Rightarrow -3d=0$$
$$\Rightarrow d=0$$

Hence,

$$b=2-4d$$
$$\Rightarrow b=2$$

Thus, $a = 1, b = 2, c = -2$ and $d = 0$

Hence, the required matrix $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Question 12:
If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in N$.

Solution:
Given: A and B are square matrices of the same order such that $AB = BA$.

To prove: $P(n): AB^n = B^n A, n \in N$

For n = 1, we have:

$$P(1): AB = BA \qquad [\text{Given}]$$
$$\Rightarrow AB^1 = B^1 A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k) = AB^k = B^k A \qquad \ldots(1)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= AB^k.B \\ &= (B^k A)B && [By\ (1)] \\ &= B^k(AB) && [\text{Associative law}] \\ &= B^k(BA) && [AB = BA\ (\text{Given})] \\ &= (B^k B)A && [\text{Associative law}] \\ &= B^{k+1}A \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A, n \in N$

Now, we have to prove that $(AB)^n = A^n B^n$ for all $n \in N$
For $n = 1$, we have:

$$(AB)^1 = A^1 B^1 = AB$$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$$(AB)^k = A^k B^k \qquad \ldots(2)$$

Now, we prove that the result is true for $n=k+1$.

$$\begin{aligned} AB^{k+1} &= (AB)^k.(AB) \\ &= (A^k B^k).(AB) && [\text{By } (2)] \\ &= A^k (B^k A) B && [\text{Associative law}] \\ &= A^k (AB^k) B && [AB^n = B^n A, n \in N] \\ &= (A^k A).(B^k B) && [\text{Associative law}] \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n, n \in N$

Question 13:

If $A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then,

(A) $1+\alpha^2+\beta\gamma=0$ (B) $1-\alpha^2+\beta\gamma=0$

(C) $1-\alpha^2-\beta\gamma=0$ (D) $1+\alpha^2-\beta\gamma=0$

Solution:

It is given that $A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Therefore,

$$\begin{aligned} A^2 &= A.A \\ &= \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \\ &= \begin{pmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{pmatrix} \\ &= \begin{pmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{pmatrix} \end{aligned}$$

Now, $A^2 = I$

Hence,

$$\begin{pmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have:

$$\begin{aligned} &\alpha^2+\beta\gamma = 1 \\ &\Rightarrow \alpha^2+\beta\gamma-1=0 \\ &\Rightarrow 1-\alpha^2-\beta\gamma=0 \end{aligned}$$

Thus, the correct option is C.

Question 14:

If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Solution:

If the matrix A is both symmetric and skew symmetric, then

$A' = A$ and $A' = -A$

Hence,

$$\begin{aligned} &\Rightarrow A = -A \\ &\Rightarrow A + A = 0 \\ &\Rightarrow 2A = 0 \\ &\Rightarrow A = 0 \end{aligned}$$

Therefore, A is a zero matrix.

Thus, the correct option is B.

Question 15:

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) $I - A$ (C) I (D) $3A$

Solution:

It is given that A is a square matrix such that $A^2 = A$.

Now,

$$\begin{aligned}(I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\ &= I + A^2.A + 3A + 3A^2 - 7A \\ &= I + A.A + 3A + 3A - 7A \quad \left[\because A^2 = A\right] \\ &= I + A^2 - A \\ &= I + A - A \quad \left[\because A^2 = A\right] \\ &= I\end{aligned}$$

Hence,

$$(I + A)^3 - 7A = I$$

Thus, the correct option is C.